Theoretical calculation and experimental measurement on production cross section of Z boson in μ-μ channel with Monte Carlo simulated data

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Summer Research Fellowship (SRF) 2020 for Science Students
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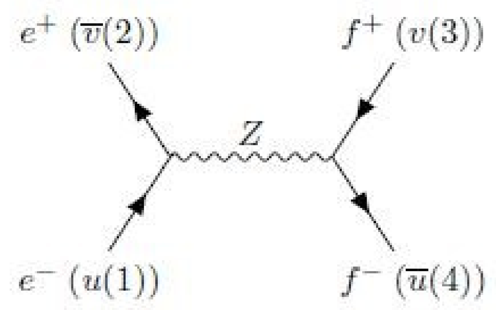
Abstract

Basics of Quantum field theory was discussed in this research. We proposed a way of calculating the production cross section of Z-boson in tree-level Feynman Diagrams using the existing Dirac Feynman Rules. Results are then compared with colliding data in Monte Carlo Event generator available in PYTHIA 8. Consistent results are attained.

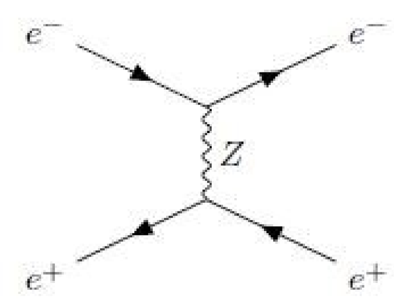
Methods

The aim for this section is to find out the cross-section for the process $e^+e^- \to Z \to e^+e^-$ and $\to \mu^+\mu^-$.

We start from the Feynman diagrams.



There is another possible Feynman diagram for $e^+e^- \to Z \to e^+e^-$ but not $\mu^+\mu^-$.



Obviously, these processes are mediated by the weak force carrier Z boson, thus we have to use the corresponding weak neutral current vertex factor and propagator.

To simplify things up, we will neglect the mass of electrons and muons, because otherwise this process will have a much smaller cross-section compared to QED processes. This simplification is justified as Z boson is much heavier than electrons and muons.

On the other hand, we will consider the Z vertex factor:

$$\frac{-ig_Z}{2}\gamma^\mu(c_V^f - c_A^f\gamma^5)$$

from now on we take $c_V - c_A\gamma^5 = \Gamma$, so new form will be

$$\frac{-ig_Z}{2}\gamma^\mu\Gamma$$

We first find out the matrix element for the s-channel diagram. By Feynman rules we have:

$$\int \frac{d^4q}{(2\pi)^4}(2\pi)^8\delta(q-p_1-p_2)\delta(q-p_3-p_4)\,\overline{u}(4)\left(\frac{-ig}{2}\gamma^\mu\Gamma\right)v(3)\left[\frac{-i(g_{\mu\nu}-q_\mu q_\nu/M^2)}{q^2-M^2}\right]\overline{v}(2)\left(\frac{-ig}{2}\gamma^\mu\Gamma\right)u(1)$$

$$\mathcal{M}_s = \frac{g_Z^2}{4}\left[\overline{u}(4)\gamma^\mu\Gamma v(3)\right]\frac{g_{\mu\nu}-q_\mu q_\nu/M^2}{q^2-M^2}\left[\overline{v}(2)\gamma^\nu\Gamma u(1)\right]$$

Note that the term with the factor $q_\mu q_\nu/M^2$ vanishes if we neglect the mass of electron and muon, as for example,

$$\overline{u}(4)\not{q}\Gamma v(3) = \overline{u}(4)\not{p}_4 X + X'\not{p}_3 v(3) = 0$$

by the massless Dirac equation $\gamma^\mu p_\mu v = 0$ and its adjoint $\overline{u}\gamma^\mu p_\mu = 0$. ($X, X'$ are some finite terms.)

Hence, the norm square of $\mathcal{M}_s$ is:

$$|\mathcal{M}_s|^2 = \frac{g_Z^4}{16(q^2-M^2)^2}[\bar{u}(4)\gamma^\mu\Gamma v(3)][\bar{v}(2)\gamma_\nu\Gamma u(1)][\bar{u}(4)\gamma^\mu\Gamma v(3)]^*[\bar{v}(2)\gamma_\nu\Gamma u(1)]^*$$

where the asterisk represent complex conjugate.

Typically, if the process has specific spins and polarization configurations, we can insert the appropriate spinors and/or ϵ to work out $\mathcal{M}$ and $|\mathcal{M}|^2$. But this is more like a subset of a real world experiment: specfic spins are not favoured over any other configuration—we just do experiment with a beam of initial particles that have random spins. Therefore, what we can do is to compute the expected value of $|\mathcal{M}|^2$ which is by averaging over all initial spins and summing all final spins .

To proceed, we will employ the Casimir's Trick to reduce the complexity of the calculations:

$$\sum_{\text{all spin}}[\bar{u}(a)\Gamma_1 u(b)][\bar{u}(a)\Gamma_2 u(b)]^* = Tr[\Gamma_1(\not{p}_b + m_b c)\bar{\Gamma}_2(\not{p}_a + m_a c)]$$

It is a very useful tool since we can get rid of the spinors, and once we do the summation over spins, the computations are trim down into trace identities of some γ matrices.

So, for electron-positron pair scattering via Z boson at high energy, we have:

$$\langle|\mathcal{M}|^2\rangle = \langle|\mathcal{M}_1 - \mathcal{M}_2|^2\rangle = \langle|\mathcal{M}_1|^2\rangle - 2Re(\langle\mathcal{M}_1^*\mathcal{M}_2\rangle) + \langle|\mathcal{M}_2|^2\rangle$$

$$\begin{aligned}\langle|\mathcal{M}|^2\rangle = &\frac{1}{2}\frac{g^4}{(q_1^2-M^2)^2}\{(c_V^2+c_A^2)^2[(p_1\cdot p_3)(p_2\cdot p_4)+(p_1\cdot p_4)(p_2\cdot p_3)] + 4c_V^2c_A^2[(p_1\cdot p_3)(p_2\cdot p_4)-(p_1\cdot p_4)(p_2\cdot p_3)]\}\\ &+\frac{1}{2}\frac{g^4}{(q_2^2-M^2)^2}\{(c_V^2+c_A^2)^2[(p_1\cdot p_3)(p_2\cdot p_4)+(p_3\cdot p_4)(p_2\cdot p_1)] + 4c_V^2c_A^2[(p_1\cdot p_3)(p_2\cdot p_4)-(p_3\cdot p_4)(p_2\cdot p_1)]\}\\ &+\frac{g^4}{2(q_1^2-M^2)(q_2^2-M_2)}(c_V^4+6c_V^2c_A^2+c_A^4)(p_2\cdot p_4)(p_1\cdot p_3)\end{aligned}$$

$$\begin{aligned}\langle|\mathcal{M}|^2\rangle = &\left(\frac{g^2E^2}{(2E)^2-M^2}\right)^2[(c_V^2+c_A^2)^2(1+\cos^2\theta) - 8c_V^2c_A^2\cos\theta]\\ &+\frac{g^4}{2}\frac{1}{(4E^2\sin^2(\theta/2)+M^2)^2}[(c_V^2+c_A^2)^2(E^4(1-\cos\theta)^2+4E^4)+4c_V^2c_A^2(E^4(1-\cos\theta)^2-4E^4)]\\ &-\frac{g^4(c_V^4+6c_V^2c_A^2+c_A^4)}{2((2E)^2-M^2)(4E^2\sin^2(\theta/2)+M^2)}(E^4(1-\cos\theta)^2)\end{aligned}$$

Theoretical results

We give the cross section for $e^+e^- \to \mu^+\mu^-$. The calculation for $e^+e^- \to \mu^+\mu^-$ is similar. Suppose CM frame energy $2E = 500GeV$. Neglect electron and muon mass. Neglect the running constant problem and simply take $g_Z = 0.718$, $c_A = -1/2$, $c_V = -1/2 + 2\sin^2\theta_W = -0.0372$ so $c_A^2 + c_V^2 \approx 1/4$. Take $M = 91.2GeV$. From Golden Rule we can derive the cross section:

$$\begin{aligned}\frac{d\sigma}{d\Omega} &= \left(\frac{1}{8\pi}\right)^2\frac{|\boldsymbol{p_f}|}{|\boldsymbol{p_i}|}\frac{|\mathcal{M}|^2}{(2E)^2}\\ \sigma &= \frac{1}{(16\pi E)^2}\int d\Omega|\mathcal{M}|^2\\ |\mathcal{M}|^2 &= \left(\frac{g^2E^2}{(2E)^2-M^2}\right)^2((c_V^2+c_A^2)^2(1+\cos^2\theta) - 8c_V^2c_A^2\cos\theta)\\ &\approx \left(\frac{g^2E^2}{(2E)^2-M^2}\right)^2((1+\cos^2\theta)/16 - 8c_V^2c_A^2\cos\theta)\\ \sigma &= \left(\frac{g^2E}{(16\pi((2E)^2-M^2)}\right)^2 2\pi\int_0^\pi \frac{1+\cos^2\theta}{16} - 8c_V^2c_A^2\cos\theta\\ &= \left(\frac{g^2E}{(16\pi((2E)^2-M^2)}\right)^2\frac{\pi}{8}\left(\frac{3}{2}\pi\right)\\ &= \frac{3g^4E^2}{16^3(4E^2-M^2)^2}\\ &\approx 8.33\times10^{-10}GeV^{-2}\end{aligned}$$

which equals a cross section of 0.324 pico-barn.

To turn to MCEG and experimental data, we use PYTHIA8:

```
#include "Pythia8/Pythia.h"
using namespace Pythia8;

int main(int argc, char* argv[]) {
  Pythia pythia;
  pythia.readFile(argv[1]);

  pythia.init();

  int n;
  for(int i=0; i<50000;++i){
    pythia.next();
    for (int j = 0; j<pythia.event.size();++j){
      if(pythia.event[j].id() == 11 || pythia.event[j].id() == -11){
    n++;
      }
    }
  }
  pythia.stat();
  cout << "##############################" <<endl;
  return 0;
}
```

```
Beams:idA = 11
Beams:idB = -11
Beams:eCM = 500
WeakZ0:gmZmode = 2
WeakBosonExchange:ff2ff(t:gmZ) = on
Next:numberShowEvent = 8
23:onIfAny = 13
```

After ~50000 events, we have filtered the useful information and store them in an out file:

```
 no        id  name            status     mothers   daughters     colours      p_x        p_y        p_z         e          m
  0        90  (system)          -11     0     0     0     0     0     0     0.000      0.000      0.000    500.000    500.000
  1        11  (e-)              -12     0     0    21     0     0     0     0.000      0.000    250.000    250.000      0.001
  2       -11  (e+)              -12     0     0    22     0     0     0     0.000      0.000   -250.000    250.000      0.001
  3        11  (e-)              -21     6     6     5     0     0     0     0.000      0.000    249.902    249.902      0.000
  4       -11  (e+)              -21     7     0     5     0     0     0     0.000      0.000     -8.790      8.790      0.000
  5        23  (Z0)              -22     3     4     8     8     0     0     0.000      0.000    241.112    258.692     93.739
  6        11  (e-)              -42    10     0     3     3     0     0    -0.000      0.000    249.902    249.902      0.000
  7       -11  (e+)              -41    11    11     9     4     0     0     0.000     -0.000   -249.804    249.804      0.000
  8        23  (Z0)              -44     5     5    12    12     0     0   -16.867     27.892    238.898    258.692     93.739
  9        22  (gamma)           -43     7     0    13    13     0     0    16.867    -27.892   -238.799    241.014      0.000
 10        11  (e-)              -41    15    15    14     6     0     0     0.000      0.000    250.000    250.000      0.000
 11       -11  (e+)              -42    16     0     7     7     0     0    -0.000      0.000   -249.804    249.804      0.000
 12        23  (Z0)              -44     8     8    17    17     0     0   -16.868     27.895    238.898    258.692     93.739
 13        22  (gamma)           -44     9     9    18    18     0     0    16.867    -27.892   -238.799    241.013      0.000
 14        22  (gamma)           -43    10     0    19    19     0     0     0.001     -0.003      0.098      0.098      0.000
 15        11  (e-)              -42    21    21    10    10     0     0    -0.000      0.000    250.000    250.000      0.000
 16       -11  (e+)              -41    22    22    20    11     0     0     0.000     -0.000   -250.000    250.000      0.000
 17        23  (Z0)              -44    12    12    23    23     0     0   -16.868     27.895    238.897    258.692     93.739
 18        22  (gamma)           -44    13    13    24    24     0     0    16.868    -27.893   -238.799    241.014      0.000
 19        22  (gamma)           -44    14    14    25    25     0     0     0.001     -0.003      0.098      0.098      0.000
 20        22  (gamma)           -43    16     0    26    26     0     0    -0.002      0.001     -0.196      0.196      0.000
 21        11  (e-)              -61     1     0    15    15     0     0    -0.000      0.000    250.000    250.000      0.000
 22       -11  (e+)              -61     2     0    16    16     0     0     0.000      0.000   -250.000    250.000      0.000
 23        23  (Z0)              -62    17    17    29    30     0     0   -16.868     27.895    238.897    258.692     93.739
 24        22  gamma              62    18    18     0     0     0     0    16.868    -27.893   -238.799    241.014      0.000
 25        22  gamma              62    19    19     0     0     0     0     0.001     -0.003      0.098      0.098      0.000
 26        22  gamma              62    20    20     0     0     0     0    -0.002      0.001     -0.196      0.196      0.000
 27        22  gamma              63     1     0     0     0     0     0     0.000      0.000      0.000      0.000      0.000
 28        22  gamma              63     2     0     0     0     0     0     0.000      0.000     -0.000      0.000      0.000
 29        13  mu-                23    23     0     0     0     0     0   -25.613     46.095    231.110    237.050      0.106
 30       -13  mu+                23    23     0     0     0     0     0     8.745    -18.200      7.787     21.642      0.106
                                   Charge sum:  0.000           Momentum sum:    -0.000     -0.000     -0.000    500.000    500.000

-------  End PYTHIA Event Listing  -----------------------------------------------------------------------

-------  PYTHIA Event and Cross Section Statistics  -------------------------------------------------*
                                                                                                     |
 Subprocess                                    Code |            Number of events       |      sigma +- delta    |
                                                    |       Tried   Selected   Accepted |     (estimated) (mb)   |
                                                    |                                   |                        |
-----------------------------------------------------------------------------------------------------|
                                                    |                                   |                        |
 f fbar -> gamma*/Z0                            221 |        4474        500        500 |   4.030e-10  9.692e-12 |
                                                    |                                   |                        |
 sum                                                |        4474        500        500 |   4.030e-10  9.692e-12 |
                                                                                                     |
-------  End PYTHIA Event and Cross Section Statistics  ---------------------------------------------*
```

From the simulation we found out that the experimental cross section of the t-channel scattering of lepton pairs to Z boson is roughly 0.4 pb, in the same magnitude as the theoretical calculations.

Acknowledgement

This Summer Research Fellowship is supervised by Dr Yanjun Tu, with the assistance of Ho Derek Yeung from the University of Oxford, Cheuk Nga Athena Ng from the King's College London, as well as Li Pak Polly Yi and Junyi Yang from HKU.

Together we have written an introductory QFT reports on the basics in calculating cross section of elementary tree-level processes.

An Introduction to Quantum Field Theory

Cross section calculation of Z^0 boson production

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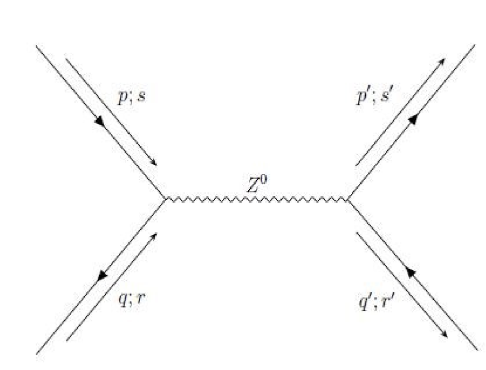

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