Summer Research Fellowship (SRF) 2020
for Science Students

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Causality of Viscous Dark Fluids

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Abstract

Dark fluids are a set of models proposed as an explanation for both dark matter and dark energy, which are both outstanding problems in physics. In this project, various dark fluid models are analyzed with Israel-Stewart theory, a theory of relativistic fluid dynamics, with the goal of determining their causality. It is found that all dark fluid models considered in this project are acausal. Conclusions drawn from these models are therefore unreliable, and further work is needed to develop a causal dark fluid model.

1. Introduction

1.1 Relativity

When velocities are close to the speed of light c, or when the gravitational field becomes large, observed phenomena deviate greatly from the Newtonian predictions. A new theory, general relativity, was necessary to explain these phenomena.

General relativity describes space and time as a metric $g_{\mu\nu}$ whose curvature is determined by the energy-momentum tensor $T_{\mu\nu}$ of the objects in the spacetime according to the Einstein field equation:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu} \qquad (1)$$

The metric signature (–,+,+,+) will be used throughout this poster.

1.2 Relativistic fluid dynamics

For a viscous fluid with bulk viscosity only, $T_{\mu\nu}$ can be expressed as

$$T^{\mu\nu} = \mathcal{E}u^\mu u^\nu + \mathcal{P}\Delta^{\mu\nu} + 2Q^{(\mu}u^{\nu)} + \mathcal{T}^{\mu\nu} \qquad (2)$$

where $\Delta_{\mu\nu} = u_\mu u_\nu + g_{\mu\nu}$.

In relativistic fluid dynamics, one hopes to construct a theory using only temperature T, chemical potential μ, and 4-velocity u^μ, as these are quantities used thermodynamics. However, such theories have produced unphysical predictions, such as acausality and instability of its thermal equilibrium state.

Israel-Stewart theory addresses this problem by introducing extra variables, e.g. the bulk relaxation time τ_Π, and is characterized by the following equations:

$$u^\mu \nabla_\mu \varepsilon + (\varepsilon + P + \Pi)\nabla_\mu u^\mu = 0 \quad (3)$$

$$(\varepsilon + P + \Pi)u^\mu \nabla_\mu u^\nu + \left.\frac{\partial P}{\partial \varepsilon}\right|_n \Delta^{\mu\nu}\nabla_\mu \varepsilon + \left.\frac{\partial P}{\partial n}\right|_\varepsilon \Delta^{\mu\nu}\nabla_\mu n + \Delta^{\mu\nu}\nabla_\mu \Pi = 0 \quad (4)$$

$$\tau_\Pi u^\mu \nabla_\mu \Pi + \Pi + \lambda \Pi^2 + \zeta \nabla_\mu u^\mu = 0 \quad (5)$$

$$u^\mu \nabla n + n \nabla_\mu u^\mu = 0 \quad (6)$$

where ζ is the bulk viscosity and $\lambda = \lambda(\varepsilon, n)$ is a transport coefficient.

In conjunction with equation (1), these equations constitute the generalized Einstein-Israel-Stewart (EIS) theory, a theory of fluid dynamics in a curved spacetime.

2. Discussion

2.1 Causality

The causality conditions for the generalized EIS theory has been solved in the full nonlinear regime in [1]. Causality is obeyed under the following conditions:

$$\frac{\zeta}{\tau_\Pi(\varepsilon + P + \Pi)} + \left.\frac{\partial P}{\partial \varepsilon}\right|_n + \left.\frac{\partial P}{\partial n}\right|_\varepsilon \frac{n}{\varepsilon} + P + \Pi \geq 0 \quad (7)$$

$$\left(\frac{\zeta}{\tau_\Pi} + n\left.\frac{\partial P}{\partial n}\right|_\varepsilon\right)\frac{1}{\varepsilon + P + \Pi} \leq 1 - \left.\frac{\partial P}{\partial \varepsilon}\right|_n \quad (8)$$

2.2 Dark Fluid Models

The first dark fluid model is the one discussed in [2]. The energy-momentum tensor for the dark fluid considered is of the form

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu + P\Delta^{\mu\nu} - \Pi \Delta^{\mu\nu}\nabla_\alpha u^\alpha \qquad (9)$$

By performing the transformation

$$P \to P - 3\Pi\frac{\dot{a}}{a} \qquad (10)$$

on the standard ideal fluid equations, one can see that the effect of the bulk scalar is to act as a negative pressure, and leads to the following solutions to the Friedmann equations:

$$a(t) = a_0 e^{Ht} \quad (11)$$

$$P = \sqrt{24\pi G \varepsilon \Pi^2} - \varepsilon \quad (12)$$

$$H^2 = \frac{8\pi}{3}G\varepsilon \quad (13)$$

where $a(t)$ is the scale factor used in the FLRW metric and (12) is the equation of state. This could potentially recover the predictions made by the addition of a cosmological constant to equation (1).

However, since the relaxation time of the bulk viscosity is not considered in this model, this implies $\tau_\Pi = 0$. As τ_Π goes to 0, the terms on the left side of equation (8) goes to infinity.

$\tau_\Pi = 0$ reduces the generalized EIS equations to the relativistic Navier-Stokes equations, which are acausal according to the causality conditions laid down in equations (7) and (8) above.

Cont'd 2.2

The second dark fluid model considered is in [3]. The effective pressure generated by the bulk viscosity would be

$$P = -3\alpha\varepsilon^m H \qquad (14)$$

for the Hubble parameter H and constants α and m fitted to the data.

Linear perturbations of this model act differently from the ΛCDM model. The viscosity mainly causes the density perturbations to quickly die down, especially at late times when such an effect becomes significant. This slows the growth in "normal" matter, and causes fluctuations in the gravitational potential to decay quickly. These effects can be seen on the large-scale CMB spectrum, weak lensing, and CMB-galaxy cross correlations.

However, this faces the same problem as the previous dark fluid model. The bulk relaxation time is once again not considered, implying the equations this model obeys are the relativistic Navier-Stokes equations, which are acausal. Further investigation will be required to modify this model into a form that obeys causality, at which point it may reduce to the ΛCDM model.

3. Conclusion

The dark fluid models considered in this project do not obey the causality conditions of the generalized Einstein-Israel Stewart equations due to the neglect of the bulk relaxation time. This violates one of the causality conditions of the generalized EIS equations, and therefore the cosmological conclusions made from these models cannot be trusted. Further work will be required to create causal versions of these models.

4. Acknowledgements

This project was originally intended for the Overseas Research Fellowship, and is done with much assistance from Prof. Jorge Noronha from the University of Illinois at Urbana-Champaign.

5. References

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