

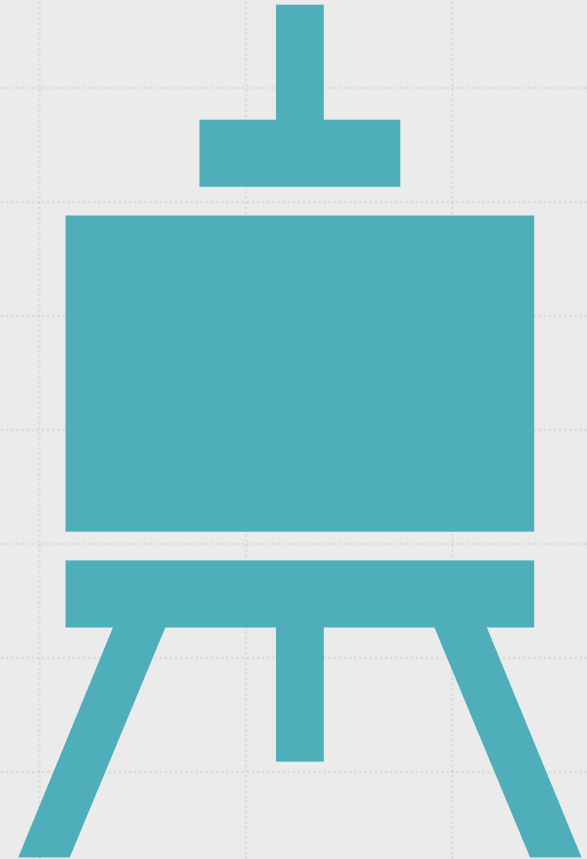


SRF (Summer Research Fellowship) Sharing Session

Wang Linjing
2023 SRF Participant, Major

Process for your SRF

- Find your supervisor
- Settle down your project
- Write a research proposal
- Selection interview
- Begin the research work
- Conclude your research results
- Final presentation



How to find your supervisor?

- Option 1: Approaching Your Course Professor
- Option 2: SRF website:
<https://www.scifac.hku.hk/current/ug/el/research/srf-orf>

The screenshot shows the HKU Science website header with the logo and navigation menu. The main heading is "SUMMER RESEARCH FELLOWSHIP (SRF) & OVERSEAS RESEARCH FELLOWSHIP (ORF) SCHEMES FOR SCIENCE STUDENTS". Below this, a paragraph explains the schemes: "Since 2007, the Faculty of Science has implemented two schemes: **Summer Research Fellowship (SRF)** & **Overseas Research Fellowship (ORF)**. The ORF Scheme provides financial support to students to go to overseas laboratories or institutes to work on research projects, while the SRF Scheme supports students to do research at a supervisor's laboratory in the Faculty of Science."

Four navigation tiles are visible:

- Introduction of SRF & ORF**: Accompanied by a blue icon of a document with a list.
- Research Opportunities from Science School / Departments**: Accompanied by a yellow icon of a head with a brain and a network diagram. This tile is highlighted with a yellow brushstroke.
- Application Details**: Accompanied by a blue icon of a calendar with a pencil.
- Poster Presentation & Research Colloquy of SRF/ORF/URFP**: Accompanied by a blue icon of a share symbol.

A "TOP" button is located in the bottom right corner of the page.

Computational Mathematics

Research Opportunities from Science School / Departments

Research opportunities are available from the Science School and Departments for HKU Science undergraduate students. Students are encouraged to visit the following websites to know more about teachers' research interests and approach them for research opportunities.

- School of Biological Sciences:
<https://www.biosch.hku.hk/about-us/our-staff>
- Department of Chemistry:
http://www.chemistry.hku.hk/pro_pgs_supervisors.php
- Department of Earth Sciences:
<https://www.earthsciences.hku.hk/current-students/undergraduate-students/research-opening-opportunities>
- Department of Mathematics:
<http://www.math.hku.hk>
(at 'Research Groups' under the 'Research' section)
- Department of Physics:
<http://www.physics.hku.hk/students/Summer/Summer>
- Department of Statistics & Actuarial Science:
<http://www.saasweb.hku.hk/research/research.php>

Members

- **Prof. W.K. Ching**
Mathematical modeling, applied computing, optimization
- **Prof. G. Han**
Coding and information theory
- **Prof. M. Ng**
Applied and Computational Mathematics, Artificial Intelligence and Machine Learning, Data and Imaging Sciences and Scientific Computing
- **Prof. X. Yuan**
Scientific computing, management science
- **Dr. G. Li**
Numerical analysis, scientific computing
- **Dr. Z. Zhang**
Scientific computation, biomechanics
- **Dr. K. Cai**, Postdoctoral Fellow
Coding theory, combinatorics
- **Dr. Z. Wu**, Postdoctoral Fellow
Scientific computing, mainly in uncertainty quantification, model reduction, quantum mechanics
- **Dr. Y. Xu**, Postdoctoral Fellow
Algebraic coding theory

Chat with your potential supervisor to get research idea

Or talk to them about your rough ideas...

Activities

Research Proposal

- After reading given materials, consider the following points:

1. **Why this research is important?**

2. **How do you plan to conduct it?**

Eg. Literature review? Experiment?

How is the design?

3. **What do you need?**

Eg. Relevant literatures,
coding language, other softwares

4. **Time/Effort allocation**

Eg. How often to meet supervisor?

Group work or Individual work



Introduce your research proposal and express your passionate during the interview.

During the research work

- Have to face intractable problems:

Accept the fact that there will always be problems that you can not solve right now. Be patient and don't give up.

Some tips may help:

- Keep in contact with your supervisor :

When facing research challenges, discussing them with your supervisor can provide valuable assistance, boost your confidence, and offer new insights to guide your progress.



During the research work

- Record your process :

You will have better control about the whole progress and be more clear about how to spend these weeks. Also, by reviewing, you will be proud of what you have already done.

- Try it tomorrow :

Sometimes sleeping will give unexpected insights. Task a break is useful and necessary.



During the research work

- Do not use FAKE data :

Academic integrity is more important than achievements. The purpose of SRF is to give us a chance to try what research is like.

- Mistakes could be creative!

Keep open-minded to the mistake you made. Sometimes, it may provide you with more idea. But do not forget to fix it.



Final step:

Poster : include introduction, methods, results, conclusions and references. Try to make it attractive.

Synopsis: include more details comparing with the poster. Make it a summarize of the whole project.

Presentation: Great chance to let more people know what you have done. Talking could give you more inspiration.

Common types of Integral Inequalities with analogues in discrete cases

Opial's inequality and Gronwall-Bellman inequality

Wang Linjing

Supervisor: Prof Wing Sum Cheung



Abstract

This research delves into the captivating realm of Analytic Inequalities, with a particular focus on Opial type and Gronwall-Bellman type inequalities. These inequalities hold substantial significance in mathematical analysis, offering valuable insights into the relationships between quantities. The study traces the historical development of Analytic Inequalities, highlighting the contributions of renowned mathematicians. It explores integral inequalities, with an emphasis on Opial type inequalities, and presents a comprehensive literature review encompassing original proofs and variations. The research further extends Opial inequalities to higher order derivatives, establishing new results and uncovering their implications. Discrete versions of Opial inequalities, such as Leusin's inequality and Wong's inequality, are explored, showcasing their discrete counterparts. Additionally, the Gronwall-Bellman inequality is examined, and generalizations and connections with Opial inequalities are investigated.

Introduction

Within the vast realm of mathematics, the significance of relations, whether equalities or inequalities, cannot be overstated. It is evident that the likelihood of two quantities being unequal far surpasses the probability of their equality. Consequently, inequalities assume a more profound and influential role in mathematical analysis, making the study of Analytic Inequalities an intriguing and captivating topic.

The exploration of inequalities as a branch of mathematics traces its roots back to the early 19th century, when visionary mathematicians such as Gauss, Cauchy, and Cebyshev laid the theoretical foundations for approximation methods. Since then, the study of inequalities has flourished, displaying an inherent beauty and demonstrating exceptional depth and breadth in its applications across various disciplines, including Physics, Biology, Statistics, and Engineering. Consequently, it has emerged as a dynamic and active research direction within mathematics, offering fundamental tools to unveil qualitative and quantitative properties of solutions to differential and integral equations, which lie at the heart of many contemporary scientific problems.

Traditionally, inequalities can be categorized into two broad types: integral inequalities, which involve integral of functions, and discrete inequalities, which encompass finite sums. For the purpose of this Summer Research Fellowship Scheme 2023, the focus is primarily on integral inequalities, with particular interest in Opial type and Gronwall-Bellman type inequalities.

Main Objectives

1. Original Opial's inequality (Opial's proof + Olech's proof)
2. Opial inequalities with higher order derivatives (Willett's inequality + Das's extension)
3. Discrete Opial inequalities (Leusin's inequality + Wong's inequality)
4. Gronwall-Bellman inequality

1. Original Opial's inequality

In my research, my primary focus has been on contributing to the field of Opial inequalities. This contribution includes an extensive literature review encompassing Opial's original proof as well as Olech's proof with weakened assumptions.

Opial's inequalities illustrate that: If $x(t) \in C^1([0, b])$ satisfies $x(0) = x(b) = 0$, and $x(t) > 0$ in $(0, b)$, then the following inequality holds:

$$\int_0^b |x(t)| \left| \frac{dx(t)}{dt} \right| dt \leq \frac{b}{4} \int_0^b \left(\frac{dx(t)}{dt} \right)^2 dt.$$

While for Olech's proof, the assumption that $x(t) > 0$ in $(0, b)$ has been dropped out, it is required that $x(t)$ be absolutely continuous. Further, it is also mentioned that the equality holds if and only if $x(t)$ satisfies the following condition:

$$x(t) = \begin{cases} at, & 0 \leq t \leq \frac{b}{2}, \\ a(b-t), & \frac{b}{2} \leq t \leq b. \end{cases}$$

2. Opial inequalities with higher order derivatives

Opial inequalities with higher order derivatives are of great significance in practical modeling problems, where the involvement of higher order derivatives is quite common. Therefore, it is natural to extend the classical Opial's inequality to incorporate these higher order derivatives.

The initial attempt to explore such extensions was undertaken by Willett in 1968. In this work, I not only review the existing proof but also present my own methodology to establish the following result:

Let $x(t) \in C^{(n)}([0, a])$ be such that $x^{(i)}(0) = 0$ for $0 \leq i \leq n-1$ ($n \geq 1$). Then, the following inequality holds:

$$\int_0^a |x(t)| x^{(n)}(t) dt \leq \frac{a^n}{n!} \int_0^a |x^{(n)}(t)|^2 dt.$$

Furthermore, there exists a sharper version of this theorem known as Das' Extension. According to Das' Extension, if $x(t) \in C^{(n)}([0, a])$ satisfies $x^{(i)}(0) = 0$ for $0 \leq i \leq n-1$ ($n \geq 1$), and if $x^{(n-1)}(t)$ is absolutely continuous with $\int_0^a |x^{(n-1)}(t)|^2 dt < \infty$, then the following inequality holds:

$$\int_0^a |x(t)| x^{(n)}(t) dt \leq c_n a^n \int_0^a |x^{(n)}(t)|^2 dt,$$

where $c_n = \frac{1}{n!} \left(\frac{n!}{n-1} \right)^{\frac{1}{n-1}}$.

3. Discrete Opial inequalities

In the realm of Opial inequalities, it is important to recognize that many problems or models in real-world situations are discrete in nature rather than continuous. Therefore, the

development of discrete versions of integral inequalities holds great significance. In the context of my research, I have specifically reviewed two notable discrete inequalities.

The first inequality is a discrete analogue of Olech's proof for the original Opial inequality, known as Leusin's inequality. It states that for a sequence of numbers $\{x_k\}_{k=0}^N$ satisfying $x_0 = x_N = 0$, the following inequality holds:

$$\sum_{k=1}^{N-1} |x_k \Delta x_k| \leq \frac{1}{2} \left[\frac{N+1}{2} \right] \sum_{k=1}^{N-1} |\Delta x_k|^2,$$

where Δ represents the forward difference operator and $\lfloor \cdot \rfloor$ denotes the greatest integer function.

Another prominent example in discrete form is Wong's inequality. For a non-decreasing sequence of non-negative numbers $\{x_k\}_{k=0}^N$ with $x_0 = 0$, the following inequality holds for $t \geq 1$:

$$\sum_{k=1}^t x_k^2 \Delta x_k \leq \left(t + \frac{1}{t} \right) \sum_{k=1}^t |\Delta x_k|^{t+1}.$$

These discrete versions of Opial inequalities offer valuable insights and tools for analyzing and solving problems that are inherently discrete in nature.

4. Gronwall-Bellman inequality

Upon reviewing the results of Opial inequalities, I have also summarized the proof of Gronwall-Bellman inequalities and endeavored to combine these two types of inequalities in search of potential new discoveries. Regrettably, my efforts did not yield any useful conclusions.

The Gronwall-Bellman inequality states that for real-valued continuous functions $g(t)$ and $y(t)$ defined on an interval I , assuming that $y(t)$ is differentiable in the interior of I and $y'(t) \leq g(t)y(t)$, we can derive the following inequality:

$$y(t) \leq y(t_0) e^{\int_{t_0}^t g(s) ds}.$$

This inequality provides an upper bound for the function $y(t)$ in terms of its initial value $y(t_0)$ and the integral of the function $g(t)$ over the interval $[t_0, t]$.

Conclusions

In summary, this literature review has examined the Opial's inequality, including its original proof and variations related to higher order derivatives and discrete forms. Efforts have also been made to explore potential connections with the Gronwall-Bellman inequality.

By delving into existing research and results on integral inequalities, the aim is to uncover their underlying principles, establish connections with other mathematical concepts, and identify their practical relevance in solving

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2023 for Science Students

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scientific problems. The exploration of specific types of inequalities provides valuable insights into their distinct properties, applications, and implications within the broader mathematical landscape.

Discussion and Forthcoming Research

However, it is important to acknowledge the limitations of this review. The selection of integral inequalities for study is by no means comprehensive, and most of them are based on existing literature. Consequently, the attempt to combine these inequalities have not yielded new results. Additionally, further investigation into Hardy inequalities and Poincaré inequalities may offer potential avenues for useful contributions.

Furthermore, recent research has shown that the boundaries between integral inequalities and discrete inequalities exhibit a certain degree of indeterminacy by considering the concept of "time scales". Therefore, in addition to analyzing discrete versions of inequalities and their analogues, exploring the context of time scales appears to be a promising direction for further investigation.

References

- [1] R. P. Agarwal, *Difference Equations and Inequalities*, Marcel Dekker Inc., New York, 1992.
- [2] R. P. Agarwal and P. Y. H. Ping, Remarks on the generalization of opial's inequality, *Journal of Mathematical Analysis and Applications*, to appear.
- [3] K. M. Das, An inequality similar to opial's inequality, *Proceedings of the American Mathematical Society*, 23(3):258-261, 1969.
- [4] K. Fan, O. Taussky, and J. Todd, Discrete analogues of inequalities of Wirtinger, *Monatshefte für Mathematik*, 59(1):73-90, 1955.
- [5] A. Lasota, A discrete boundary value problem, *Annales Polonici Mathematici*, 20(2):183-190, 1968.
- [6] C. Olech, A simple proof of a certain result of a opial, *Annali Polonici Mathematici*, 8(1):61-63, 1960.
- [7] Z. Opial, Sur une inégalité, *Annales Polonici Mathematici*, 8(1):29-32, 1960.
- [8] D. Willett, The existence-uniqueness theorem for an n-th order linear ordinary differential equation, *The American Mathematical Monthly*, 75(2):174-178, 1968.
- [9] J. S. W. Wong, A discrete analogue of opial's inequality, *Canadian Mathematical Bulletin*, 10(1):115-118, 1967.
- [10] H. Ye, J. Guo, and Y. Ding, A generalized gronwall inequality and its application to a fractional differential equation, *Journal of Mathematical Analysis and Applications*, 328(2):1075-1081, 2007.

Thanks for listening
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Good Luck!

